A Methodology for Model Selection in Engineering Design

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Engineering design consists of a series of stages during which a number of decisions need to be made by the designer. Since the information available to the designer is limited during initial design stages, to make these decisions and be able to proceed further in the design process, the designer needs to depict the nature, visualize the form, and predict the behavior of the product through the use of aids called models. These models guide these decisions, therefore, the designer needs to ensure the downstream validity of these decisions by constructing models with sufficient accuracy and resolution. Because higher quality and accuracy of information is most often accompanied by a higher cost for a model, determining a satisfactory level of goodness for a model is a fundamental and pervasive question in engineering. Hence, a key aspect of design model construction is deciding whether a model is appropriate for a particular design specification or evaluation, considering accuracy and cost factors. This paper presents an approach for design model construction using utility theory. Since model selection is a design decision, uncertainties in parameters and models are considered by evaluating the confidence in the selection of any model. A method for proceeding in the reverse manner to determine the required goodness of a model is also discussed. We present this research through application to a race car sway bar. [DOI: 10.1115/1.1830048]

1 Introduction

The goal of engineering design is to realize the form of a concept that satisfies the needs of a customer. This is a sequential and iterative process consisting of steps including the generation of ideas, development of concepts, embodiment of the same, detail design, and construction of a prototype.

In each stage of design, models are used to predict the result of, or guide, design specifications at a time when the design can still be changed with minimal negative impact. Because no product exists yet, available modeling information is limited. Nevertheless, designers must still construct a model that provides enough accuracy and resolution to guide and evaluate design specifications. In the case of a redesign effort, the difficulty in constructing models is decreased by the concrete information provided by the existing instance of the product. Nevertheless, the fundamental problem is the equivalent: models are constructed to predict how design changes will affect product performance. In this predictive context, model building for design is a critical and fundamental challenge for engineering design.

At a fundamental level, a model can be any abstraction of reality used to represent the nature of any system [1]. Of interest to the discussion here is the notion of an engineering design model. Though the roots of engineering models are generally scientific, engineering design models exhibit a key distinction from scientific models. Though understanding science and physics almost certainly leads to constructing better models, models need not contain direct representations of the underlying physics. The goal of a design model is not to increase scientific knowledge, but to predict actual system behavior with sufficient accuracy and resolution so design decisions can be made correctly.

Another key feature of engineering design models is that they may often exhibit poor accuracy and resolution and be based on limited information without creating uncertainty or a lack of conviction in the decision or specification for which the model is being used. Shown in Fig. 1 are three concept variants for a bicycle wheel. The extreme simplicity of this model is used to make a point. The error and ambiguity in these models leaves little uncertainty in selecting the superior concept.

During model construction, levels of abstraction are chosen, solution procedures are implemented, and results are used to make the design specifications. The modeling goal is to predict actual behavior sufficiently so that it is within some tolerance of the desired behavior. As shown in Fig. 2, error affects every stage of the design modeling process. These errors affect model truth. In a general context, truth is used to represent notions of model accuracy, resolution, and how closely the model represents the resulting reality. Errors affecting model truth influence the ability to make design specifications that produce the desired result. Nevertheless, design models can generally be constructed to sufficiently high levels of truth.

Consider the premise that a more truthful model can always be constructed. For example, to evaluate a bridge design, the bridge could be built and tested past the failure point. Though often not practical, in principle it is possible to construct the complete space of models from random guess (no model) to actual product. The "build-it-to-model-it" model is a conceptual possibility for any product design evaluation. Though these "models" are still susceptible to parameter and interpretive error, from the perspective of validating engineering design decisions, an instance of the final product is the equivalent of truth.

Because a more truthful model could always be constructed, a key design task is to decide the level of truth required for some decision. Therefore, the research in this paper focuses on the abstraction error (shown in the bold ellipse in Fig. 2). Very often the designer chooses the level of abstraction required of a model with the knowledge that a more truthful model could be constructed. The question is, "how truthful does the model need to be?"

To answer that question, we make two key observations. First, we recognize the selection of an appropriate model as a decision. The decision faced by the designer is whether to make the design decision using the current model or to make another model with better accuracy and resolution. Second, models should be selected not on the basis of resolution of various solution methods or comfort with a specific modeling technique, but on the model's overall benefit, or utility, to the designer. These observations provide for the approach of this paper. The key explorations presented here are the fundamentals of applying utility theory to the problem of

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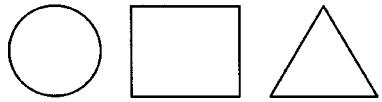

Fig. 1 Concept variants for a bicycle wheel

model selection, including the construction of model utilities and how model uncertainty relates to selecting the best model for some specification or decision. First, some brief background is presented.

2 Related Work and Background

Here, current practice for model construction in design education and industry is discussed. Research efforts in the area of design decision making and model construction are also briefly reviewed. Also included is a brief discussion of utility theory as applied to model selection.

2.1 Current Practice. Design education as it exists today stresses presenting students with increasingly advanced techniques for design and analysis. Students are taught optimization routines, finite element analysis (FEA), rapid prototyping, and other modern modeling techniques to come up with better designs and to perform design efficiently. What is lacking is the ability to decide what method to use to make any particular design decision.

For example, to analyze a pressure vessel, students have to decide whether to use basic mathematical equations to study the stress and deformations levels or to go for higher-order FEA (finite element analysis) to get a more accurate picture. The decision to choose one model over another is most often made randomly without any consideration being given to any of the factors affecting that decision. Hence, even though advanced modeling methods are available, effective utilization of these resources is poor due to a lack of ability to judge the suitability of any method or technique for a given purpose.

The approach for design model construction in industry is very similar to the method used by academics and students. Decisions regarding model construction in industry are based on the intuition and prior experience of the designer. In industry, design analysis is often decomposed based on combinations of software expertise and model form. For example, there are often solid modeling, FEA, and prototyping groups. As it is practiced in industry, the state of the art in model construction is often driven by only the resolution of various solution methods [2].

Theories for model construction are not well developed in any arena. Because there is little in the way of fundamental and mathematically based model construction approaches available for en-

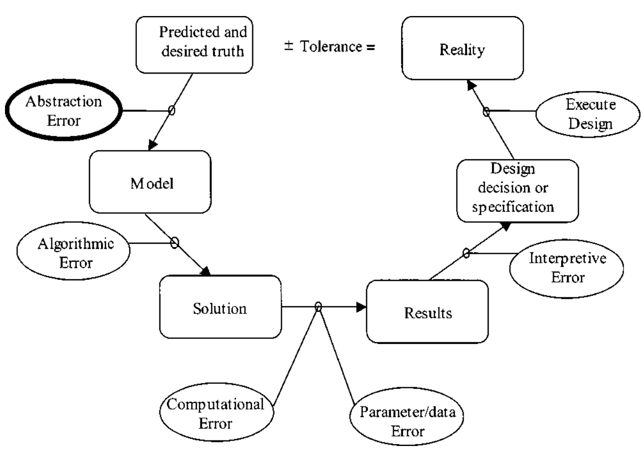

Fig. 2 Schematic showing sources of error in engineering design models

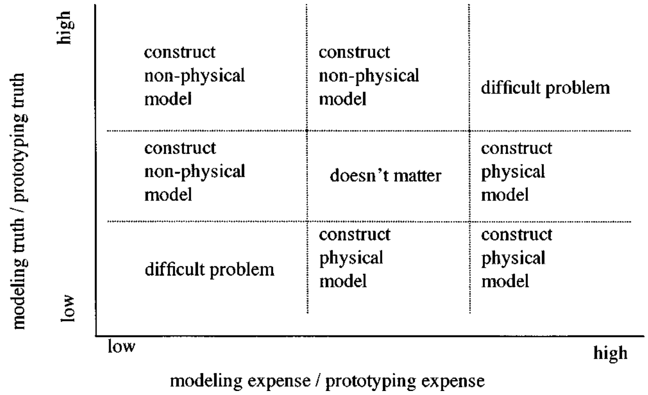

Fig. 3 Tool used for model selection guidance (adapted from [3])

gineering educators, the "what to," "how to," and "how good" type of modeling guidance taught to students is qualitative, anecdotal, and judgment based. Shown in Fig. 3 is the type of material available to guide students' modeling efforts. This simple chart provides important guidelines for students. Nevertheless, because it has limited quantitative information, it is difficult to teach and apply. These drawbacks motivate the development of a more rigorous approach for design model construction.

2.2 Related Research. The problem of selection of concepts in design is very similar to model selection because both are types of design decisions that need to be made on the basis of available scientific information and designer preferences. Thus, the related research is primarily in the areas of decision making and concept selection in engineering design. The search of methods for concept selection in design has motivated efforts to apply mathematical decision-making techniques to engineering design problems. A variety of mathematical techniques have been developed for this purpose.

Currently, the various concept selection methodologies use different procedures and measurement scales. Otto and Wood [4,5] classify concept selection methods as ordinal, interval, ratio, and extensively measurable scales. They have developed a selection methodology that allows uncertainty to be considered using different measurement scales.

Utility theory, developed by Von Neumann and Morgenstern [6], is a method for making decisions when there are a set of criteria and a number of alternatives from which to choose. This method was originally developed for economic decision making, but has been adapted to solve a variety of other problems. Keeney and Raiffa [7] present an introduction to utility theory and illustrate its application to different problems. Thurston et al. [8] introduce utility in the context of design and present an approach for finding an optimal design solution using design utility. Thurston and Carnahan [9] compare fuzzy ratings and utility analysis for use in preliminary design evaluation of multiple attributes. Otto and Antonsson [10] also discuss the application of the method of imprecision as compared to the use of utility theory for design. A method for quantifying the "house of quality" using utility theory has been presented in [11].

A majority of current research in the area of model making for engineering design addresses how to make decisions and specifications *in spite of* the uncertainty in models rather than how to construct a model that is sufficient to make a decision or specification. Fuzzy sets, interval analysis, and stochastic, probabilistic, and other methods have been applied to determine uncertainty in engineering design [4,5,10]. Most of the research in this area concerns outcome uncertainty estimation based on input parameter uncertainty. Magnusson lists the main methods for uncertainty estimation [12] as the method of moments, Monte Carlo simulation, and event-tree methods. Otto and Wood [5] present an approach for error propagation in engineering design based on the method of moments. There are not many approaches available for quantifying outcome uncertainty based on the uncertainties in the models. An approach called Bayesian Model Averaging has been used for estimating the uncertainties in outcomes due to model uncertainties [13,14]. This method uses prior probability densities of models in a specified set for determining the predictive distribution of a particular model. This method is explained with reference to a computational structural dynamics problem in [14].

An approach based on Bayesian decision theory has been applied to the assessment of computational engineering models used for design analysis by Doraiswamy et al. [15] and Doraiswamy and Krishnamurty [16]. Engineering models are assessed under conditions of uncertainty by generating required probabilities in a Bayesian tree format. This is used for selection of the best FEA model. In this method, the selection of the best analysis model is based, under conditions of uncertainty by considering the models' performance, on each design attribute and on the payoffs resulting from the outcomes.

In summary, dealing with model truth is a difficult problem. Most efforts that address model error and uncertainty are focused on parameter uncertainty, or experimental practice in which the question is how many trials (samples) of the experiment need to be run. Perhaps because of the difficulty inherent in the general model construction problem, there is limited literature in the area.

As a first step toward dealing with the general model construction problem, we apply well-established methods from decision theory to the problem of model selection. The goal (and result) is a quantitative approach to model selection that in many ways parallels the qualitative tool shown in Fig. 3. The approach presented in this paper is important as it addresses the difficult and pervasive problem of model selection. Additionally, the approach is fundamental and repeatable, as it is based on established mathematical methods, and novel, in that little prior work exists in the literature directly addressing model selection with decision theory.

The work presented in this paper is closely related to that of Doraiswamy et al. [15] and Doraiswamy and Krishnamurty [16]. In both the work of [15,16] and the work presented here, the difficulty of selecting a best model is recognized as a design decision problem and concepts form utility theory are brought to bear on the problem. In [15,16], the focus is on determining optimal parameter values for structures through FEA models. Initial model cost and truthfulness estimates are determined by running the FEA models on a reduced space of parameter values using techniques from design of experiments (DOE). In other words, the models (solid model of structure, element size, etc.) were all constructed initially and the decision was determining which model to use for a full parameter optimization problem.

This paper addresses the model selection problem at a more general level. We decide between analytical and computation models. Additionally, we wish to completely avoid the cost of any initial model construction. The approach used by [15,16] allows high-quality estimates of the expected usefulness of each model. The approach developed here does not. To address this uncertainty we will employ decision confidence methods developed by Otto and Wood [5].

2.3 Utility Theory. Recall that the selection of a model—to make discrete decisions or continuous parameter specifications—is a decision. Utility theory is a method for making decisions when there are a set of criteria and a number of alternatives from which to choose. Essentially, utility theory deals with assigning value terms to consequences of a set of action choices for making a given decision and making that decision based on these values.

Utility is the value for the usefulness of any action. For example, let $A^1, A^2, \ldots, A^m$ denote a set of action choices for a particular decision. The set of corresponding consequences can be denoted by $C^1, C^2, \ldots, C^m$. Each consequence will have some utility value that can be based on the value of some attribute, which defines that consequence.

For example, if actions A^1, A^2, and A^3 refer to selecting one of three boxes containing 10, 20, or 30 coins, then the consequences C^1, C^2, and C^3 for each of the actions can be expressed as the number of coins collected. Let C^1, C^2, and C^3 correspond to the 10, 20, and 30 coins, respectively. If we take a utility value U_{low} for the least number of coins (10) to be 0 and a value U_{high} for the highest number of coins (30) to be unity, we can find the utility value for C^2 by using some mapping between the number of coins and consequence utility. For example, $U(C^2)$ can be given as $U(C^2) = (20-10)/(30-10) = 0.67$.

In the multiattribute case where the consequences depend on more than one attribute, a single-objective utility function can be constructed to represent the utility of a consequence. The utility objective function is constructed based on the value associated with the levels of a set of attributes and the designer's preferences among this set of attributes. This means that tradeoffs between the attributes are used to determine the preference structure, and the maximum value of the objective function corresponds to the best alternative among the given set of alternatives.

For example, let $i = 1, 2 \ldots n$ refer to the set of n attributes that are relevant in making a particular decision and x_i refer to the performance level of attribute i. Let w_i and b_i represent the worst and best values for the performance level of the ith attribute. Then we can associate a value function u_i for the ith attribute such that $u_i(w_i) = 0$ and $u_i(b_i) = 1$. So for each of the alternatives we can assess the value function for each attribute corresponding to the performance level of that attribute. All of these value functions are then grouped together by assigning preferences μ_i for each attribute such that $\Sigma \mu_i = 1$. A multiattribute utility function can have many forms. The additive utility function has a form

$$U = \sum_n \mu_i u_i \tag{1}$$

where U represents the overall utility function.

The general approach explained above can be extended to make design decisions, too. For example, if a designer has to choose between a diesel and gasoline engine for a car that is being built, the usefulness of each type of engine can be evaluated on the basis of a set of attributes, which would include the size of the engine for the same power, cost required to build each type of engine, complexity of the engines, time required to build it, etc. As discussed earlier, utility theory has been applied to engineering design to solve a variety of problems, including concept selection and design optimization [9–11].

2.4 Utility Theory Applied to Model Selection. In the context of selecting an engineering design model, an action is the selection of a model from the space of available models. Note that, in practice, this space of models is discrete and finite. For example, if a simple spring is being designed to provide some deflection under load, we could model the spring using $F = kx$, an FEA model, or a physical prototype to analyze the deflection for the given force. The action choice is the selection of one of these models. The consequence corresponding to the action of selecting a model is the value of usefulness of that model for the problem at hand.

For making any design decision we can use a number of models. These models will vary from the one with the least truth to the one with the most. The simplest model will have the least value of utility for its truth with the highest value of utility for cost or time to make this model. The most truthful model will have a high value of utility for its truth, but will have comparatively higher cost and time for building it and, thus, a lower utility with respect to the effort required to construct it.

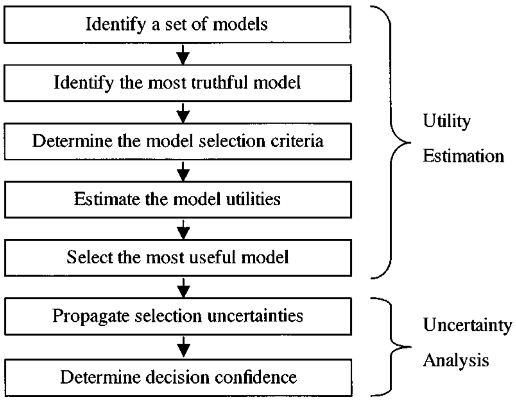

Fig. 4 Diagram showing the steps of the Model Sufficiency Estimation Technique

3 Model Sufficiency Estimation Technique

This research strives to develop a solution to the problem of model selection by using utility theory. While making the decision to select any model, the designer should also consider the effect of these uncertainties on the selection of any model. Hence, the framework for model construction in this research consists of two main parts: utility estimation and uncertainty propagation.

The flowchart shown in Fig. 4 presents the process, called the Model Sufficiency Estimation Technique, suggested for design model selection [17]. This process is the basic methodology by which a best or most useful model is selected for engineering design. The method is broken into two main stages: utility estimation and uncertainty analysis. Utility estimation is the stage wherein the value of the usefulness of each model is determined using utility theory. Once the overall utilities of each model are determined, the model having the highest value of utility is selected. The next stage is uncertainty analysis. In this stage, the uncertainties and errors in attribute values are propagated to determine the resultant uncertainty in the models. These uncertainty values are then used to evaluate the level of designer's confidence in having selected the most useful model. In Section 3.1, utility estimation is explained. Before moving to the uncertainty analysis, a utility estimation example is presented. This example is extended to explain and clarify the uncertainty analysis.

3.1 Developing Expected Model Utilities. A key need to allow the application of utility theory to model selection is developing the notion of model utility and utility functions. Here, a procedure for developing utilities for engineering design models is presented. As discussed earlier, the key factors are the ability of the model to predict a resultant behavior and the cost and effort required to make such a model. The expected values of the performance parameters affecting the decision are taken for each model along with the cost of each model. The accuracy of each model is determined by the degree of conformance of these values to values of the best model. The accuracy of any model and the cost and time required to build it determine the usefulness of the model. Hence, using the data, a single expected value for the utility of each model can be estimated. These values can then be compared to find the model with the highest utility. Following are the steps outlining the procedure for the estimation of expected utilities of design models.

Step 1: Identify Model Options. The designer has to first identify the discrete set of available models for making the specific decision.

Step 2: Identify Most Truthful Model. The model with the highest accuracy and resolution among the discrete set has to be determined. This model is termed the most truthful model and serves as a baseline for comparing the relative truth of the other models. Determining the truthfulness of a model can be a difficult task. However, here we are concerned only with the relative truthfulness of a model. Identifying the truthfulness of a model should be based on standard competency with constructing and executing the identified model. If the designer is unsure about which model *ought* (if executed with reasonable competency) to produce the most truthful result, then both can be assigned the most truthful model status. Also, in many cases, the "build-it-to-model-it" model can be included in the set of model options. Thus, this model is the most truthful model.

Step 3: Identify Model Selection Criteria. As discussed above, predicting reality accurately at a small cost is what makes a model useful to a designer. Thus, the selection criteria for a model are the error or accuracy in its prediction of component, product, or system performance and its cost. The crispness and confidence with which these selection criteria values are known may be limited at times. Thus, these selection criteria must be estimated as expected errors or accuracies and expected costs with corresponding uncertainty values.

In the formulation of the model selection problem presented here, the term model cost refers only to the cost of constructing the model, not the costs incurred if the model is inaccurate to the degree that it leads to a failed design. The costs of design failure (resulting from a poor model) are included in the method through the assignment of scaling factors to the different performance parameters predicted by the model. Thus, as the designer estimates model truthfulness, the associated uncertainties, and weights for relevant performance parameters and cost, the designer is reasoning about the cost of model construction and the cost of design failure in a structured manner. For example, if it is very important that a model predict a stress accurately to prevent catastrophic and expensive failure, then in the construction of the multiattribute utility function, the stress number produce by the model will have a relatively high value.

Step 4: Determine Model Utilities. Based on the values of the performance parameters and cost for each model, the utilities of the models can be generated. The utility function of a model is a mapping between the overall model usefulness and the error or accuracy and cost of the model. The error or accuracy of any model is defined by the degree of conformance of its predicted performance parameter values to those of the best model. Hence, we find out the utility for a model with respect to each of the performance parameters and cost. These are called single-attribute utility functions. These mappings can be given as

$$u_i = g(p_i) \tag{2}$$

Equation (2) denotes the value of usefulness associated with the level of any performance parameter, including a cost penalty, for a model. The resultant utility of the model can be given by an additive utility mapping function. This can be given by Eq. (1). U represents the overall utility function, and μ_i represents the preference for the ith performance parameter. The scaling factor quantifies the preference given by the designer for any performance parameter when compared to the other performance parameters.

Step 5: Select Most Useful Model. The model with the highest value for expected utility is determined and selected.

3.2 Example: Sway Bar Design. The method outlined above is explained here by application to a simple engineering design problem. The problem is the design of an adjustable sway bar system for a Formula Society of Automotive Engineers (FSAE) race car. The sway bar system, as seen from the rear of the car, is shown in Fig. 5. An up or down motion of the wheels causes a corresponding motion in the cantilever springs of the sway bar system. If both wheels move up the same amount, then there is no twist in the torsion spring, and thus no added stiffness to the car's ride. If the wheels move up different amounts, as

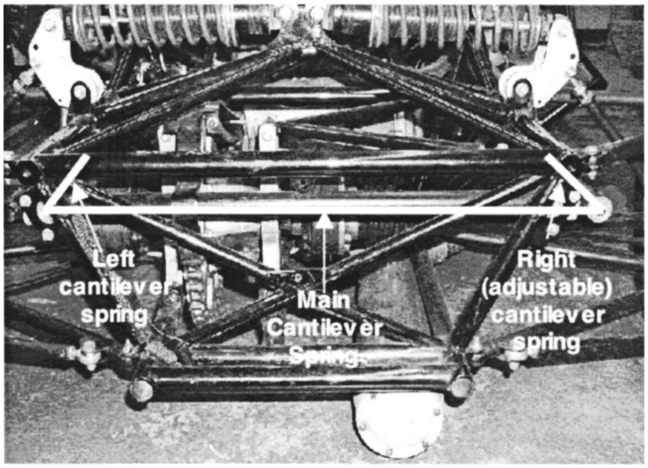

Fig. 5 View of the sway bar system (from the rear of the car)

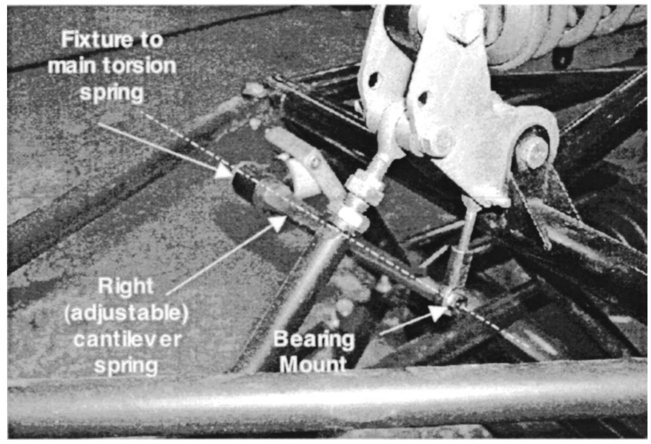

Fig. 7 Close-up view of the cantilever spring in flat position

would occur while driving through a sharp corner, then the main torsion spring is twisted. The stiffness of this torsion bar resists roll, or sway, of the car to improve car maneuverability. The total stiffness of the sway bar can be viewed as three (mechanical) springs in series. The left and right cantilever springs are deformed in bending, and the main torsion spring twists. The function of the sway bar is illustrated more clearly with the schematic in Fig. 6. The deflection of the main torsion spring is constrained at A and B so that it sees (approximately) a pure torsion.

The sway bar system is required to have a range of spring rates to tailor the race car setup to individual driving preferences. The sway bar system spring rate is adjusted by rotating the right cantilever spring. As shown in Fig. 7 (a illustration of the spring can also be seen in Fig. 11), the right cantilever spring is not symmetric about its axis of rotation. The flat position (as shown in the figure) provides the softest spring rate. When rotated 90° to the upright position, the spring is in the stiffest position. By changing the stiffness of the right cantilever spring, the overall spring rate of the sway bar system is controlled.

In order to satisfy performance criteria for the sway bar system overall, the cantilever spring has the following constraints: spring rate in flat position=34.338 N/mm (196 lb/in.), spring rate in vertical position=210.2 N/mm (1200 lb/in.), and corner stresses =1365 MPa (198 kips). The cantilever spring is 4340 steel.

All of the dimensional parameters, except the thickness of the cantilever spring, are invariable as they are constrained by mating dimensions. The original cantilever spring concept, including geometry, was generated from memory.[1] The initial value for the thickness of the sway bar is 5.56 mm (0.219 in.). Thus, the design task is to construct a model, evaluate and predict the performance of the cantilever spring, and then proceed to manufacture and usage. For this example, the design problem can be summarized as follows:

- Decision: Decide whether the proposed design can be used for the race car or the thickness of the sway bar needs to be increased for continuous use in the race car.
- Subdecision: Decide which model will be sufficient to make this decision.

Step 1: Identify Model Options. For the example here, we will consider choosing a model from a set of three manually solvable mathematical models and one FEA model. The four models, shown in Figs. 8–11, represent different levels of abstraction with respect to simplified geometry. The abstraction of Model 1, shown in Fig. 8, neglects the taper and fillets of the spring. The abstraction of Model 2, shown in Fig. 9, adds tapers in cylindrical sections shown in the previous model, but no fillets or taper in the midsection. The abstraction of Model 3, shown in Fig. 10, includes all tapers at the ends of the midsection. For each of these models, closed-form mathematical expressions of increasing complexity can be developed for design evaluation. Spring stiffness expressions were developed using Castigliano's Method. Standard

[1]A member of the student design team had seen a sway bar system similar to the example here on a different race car. The student developed the original concept from observation and memory. The fact that this is not the most rigorous method for concept generation is cursory to the discussion here.

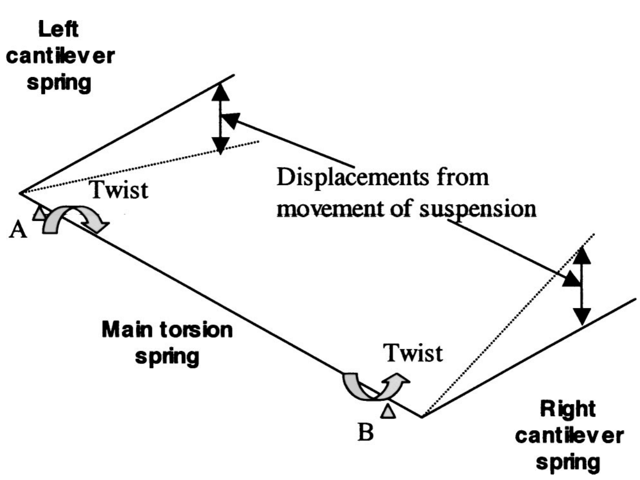

Fig. 6 Schematic showing the operation of the sway bar system

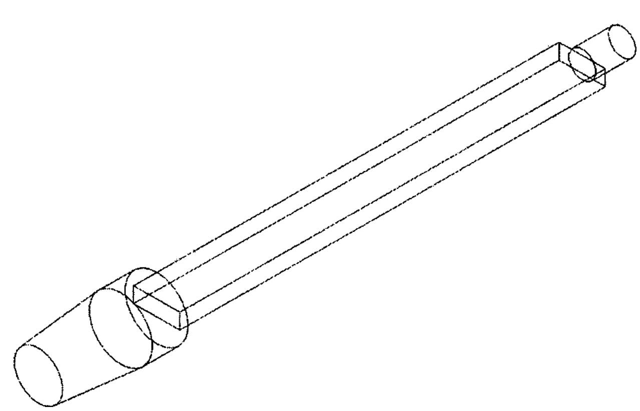

Fig. 8 Line drawing of Model 1

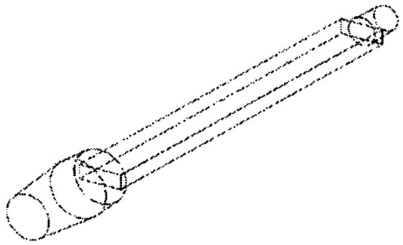

Fig. 9 Line drawing of Model 2

Table 1 Expected values of performance predicted by each model

	Model 1	Model 2	Model 3	FEA Model
Stiffness (N/mm)	31.164	32.129	35.440	35.389
Maximum stress (MPa)	1348	1306	1004	999
Number of cycles	0.81×10^3	1.12×10^3	18.40×10^3	19.38×10^3
Cost to model ($)	18	24	27	80

bending stress models were used to calculate maximum stresses. Spring life estimates were developed using stress-life fatigue models. Each of these methods is common practice. Detailed discussion of their application can be found in a standard machine design textbook such as [18]. Model 4, shown in Fig. 11, represents the geometry used to produce an FEA model. In this case, all tapers and fillets are considered.

An experimental or prototype model will not be considered here. An experimental model in the decision space reflects the ''build-it-to-model-it'' option. As the experimental model represents a high level of model truth, it can be included in the decision space to allow the designer to reason about the entire range of tradeoffs between model cost and truth in cases where the system is complex. In this case, the cantilever spring has a simple geometry, and a FEA model was deemed sufficient to represent a rational model space for the decision.

Step 2: Identify the Most Truthful Model. Of the models available, the FEA model has highest accuracy and resolution. It will be taken as the most truthful model.

Step 3: Identify Relevant Performance Parameters. The performance parameters for the cantilever spring are stiffness in the flat position, maximum stress, and maximum number of cycles before failure under this maximum stress. Thus, the performance parameters in our models are the error these models predict (as compared to the most truthful model) in these spring performance parameters and the cost to construct the model.

Here, error refers to the difference between any model's predicted performance value and that predicted by the most truthful model. The cost penalty for each model is approximated by recording construction time while performing this research. The cost of the software and computer required to perform the computational analysis were not included in the model cost.

Step 4: Determine Model Utilities. Now, different model utilities are developed. As discussed in Section 3, the utility of any model for making the specified decision depends on the accuracy of the model and the cost of making that model. The expected performance values predicted by each model are shown in Table 1.

Shown in Table 2, is the predicted performance error for the model in the column header. The error is the absolute difference in the performance when compared to the FEA model. Because our goal here is to explore the construction of utility functions for design models, we compute the actual model error rather then estimate it as discussed above. The other values are obtained as explained above.

Now, the assessment of individual attribute utility functions can be made. These can be obtained by querying the designer's pref-

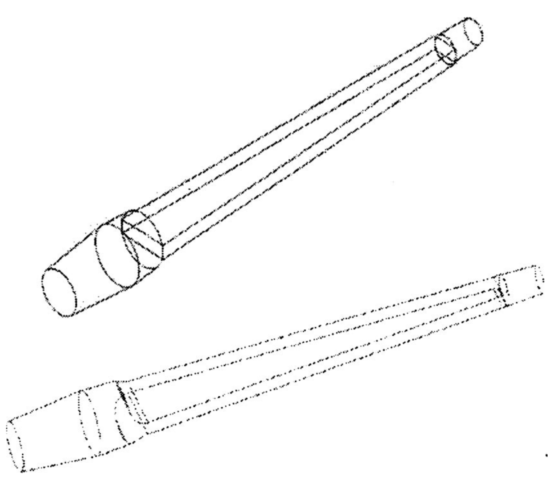

Fig. 10 Line drawing of Model 3

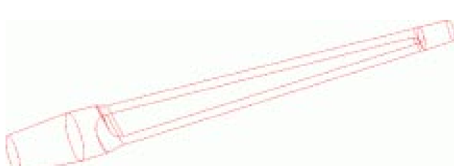

Fig. 11 Line drawing of Model 4

Table 2 Error in model performance prediction as compared to most truthful model

	Model 1	Model 2	Model 3	FEA Model
Stiffness (N/mm)	4.225	3.260	0.051	0
Maximum stress (MPa)	349	307	5	0
Number of cycles	18.58×10^3	18.26×10^3	0.98×10^3	0

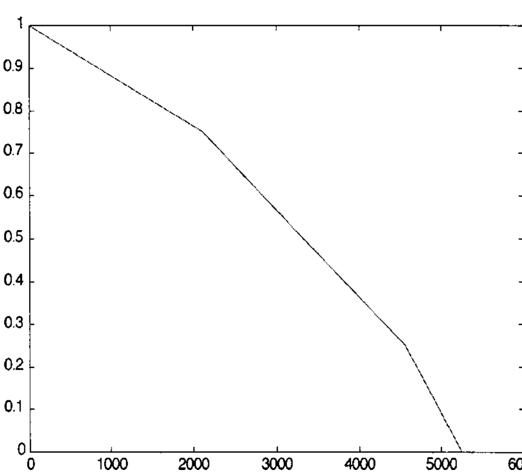

Fig. 12 Utility function for maximum stress (in MPa)

Table 3 Expected utility values

Expected utilities (Utile)	Model 1	Model 2	Model 3	FEA Model
Stiffness in flat position	0.27	0.48	0.96	1
Maximum stress	0.08	0.15	0.99	1
Number of cycles	0.05	0.08	0.97	1
Cost to model ($)	1	0.90	0.83	0

erence as explained in [7]. The attribute utility functions represent the designer's estimate of usefulness for the closeness of the value of any attribute to the values of that attribute as given by the best model. For example, the individual attribute function for the utility corresponding to the absolute values of deviation or error in the maximum stress (as given in Table 2) is shown in Fig. 12. This shows that for a model with no deviation from the most truthful model (the FEA model in this case) the utility is 1 and for any model having a difference of 360 MPa from the value given by the best model, the utility is 0. To give an example, if we want to determine the value of stress corresponding to zero utility, the designer can be asked the following question: *Considering only the maximum stress criteria, above what value of deviation from the value of stress for the best model (999 MPa) is any model not useful at all*? In this case, the value is 360 MPa. Questions can be framed similarly to obtain the data required to construct the utility functions for each of the criteria. Using such an approach, utility functions were generated for the sway bar problem. Using these utility curves, the relevant utilities for each of the model performance estimation is shown in Table 3.

To select the overall most useful model, a multiattribute utility function is constructed. The scaling constants can be obtained by considering the tradeoffs [6]. Using the notation of Eq. (1), let μ_1, μ_2, μ_3, and μ_4 refer to the scaling constants for stiffness in the flat position, stress, number of cycles, and cost, respectively. These constants can be determined by finding points of equivalent preference as described in [6] for overall model utility. In short, there are four unknowns so four equations are needed. The four equations are developed by identifying four pairs of hypothetical models (different combinations of deviations in stiffness, stress, number of cycles, and cost from the most truthful model) for which the overall preference for the two models is the same. For example, the following pair is compared for equivalence: (5.255 N/mm, 0 MPa, 40 cycles, $80)$\sim$($x$ N/mm, 360 MPa, 40 cycles, $80). In this comparison, the cost and cycle deviations are the same. The question to the designer is, given the model to the right of the equivalence sign ($\sim$), for what stiffness deviation value is the overall model utility the same as the model to the left of the equivalence sign? By asking such a question of the designer, a value of 3.330 N/mm for the difference in stiffness values was obtained as the value that would make the two pairs equivalent. Using Eq. (1) to calculate the utilities gives

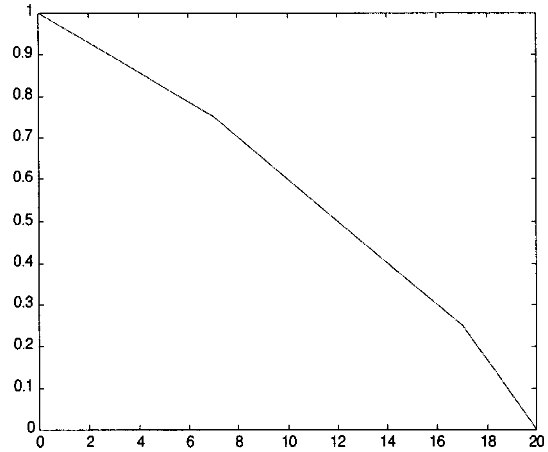

Fig. 14 Utility function for number of cycles

$$\mu_1\times(5.255)+\mu_2\times u_2(0)+\mu_3\times u_3(40)+\mu_4\times u_4(80) = \mu_1\times u_1(x)+\mu_2\times u_2(360)+\mu_3\times u_3(40)+\mu_4\times u_4(80) \quad (3)$$

Using the utility curves from Figs. 12–15 gives $\mu_2=\mu_1\times 0.5$. By repeating the exercises to develop three more relations and solving all of them, we get the scaling constants $\mu_1=0.2083$, $\mu_2=0.1667$, $\mu_3=0.1042$, and $\mu_4=0.5208$. Using Eq. (1) and the scaling constants, gives utility values for Model 1=0.5956, Model 2=0.6020, Model 3=0.8983, and Model 4=0.4792.

By observing the calculations given above, we realize that the estimation of utility values and preferences are based on the value attached to the attribute levels by the designer. Doubts about the validity of these values could arise due to the chance of designer bias [10]. These can be avoided or eliminated by including some questions to check for consistency or presenting the questions to more than one designer and taking the averages [10].

Step 5: Select Model. Considering the expected utilities, the third model is clearly the best among the four. Simply put, in spite of the error in the model, it is a better design model. This means that from a practical perspective, the FEA model is unnecessary to evaluate the fitness of the cantilever design. A review of the four

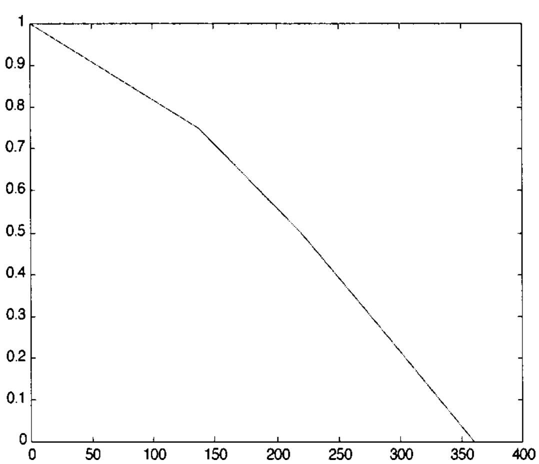

Fig. 13 Utility function for stiffness (in N/m)

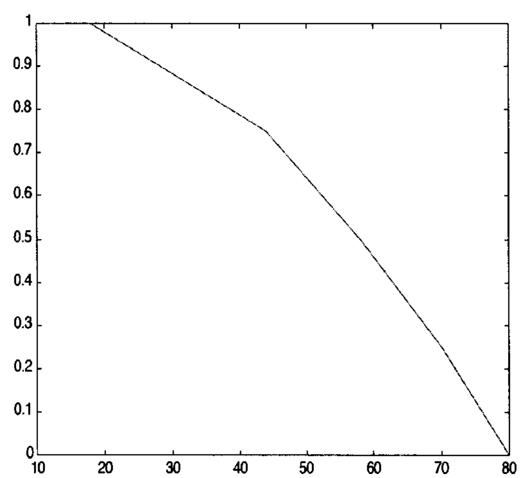

Fig. 15 Utility function for cost (in $)

Table 4 Uncertainties in performance values

	Model 1	Model 2	Model 3	Model 4
Stiffness (N/mm)	± 4.380	± 4.030	± 1.750	± 0
Maximum stress (MPa)	± 360	± 345	± 83	± 0
Number of cycles	$\pm 19\times 10^3$	$\pm 18\times 10^3$	$\pm 3\times 10^3$	± 0
Cost to model ($)	± 4	± 4	± 7.5	± 9

Table 5 Uncertainties in utility values

Utility uncertainty	Model 1	Model 2	Model 3	FEA Model
Stiffness in flat position (N/mm)	± 0.278	± 0.255	± 0.111	± 0
Maximum stress (MPa)	± 0.333	± 0.320	± 0.076	± 0
Number of cycles	± 0.316	± 0.300	± 0.050	± 0
Cost to model ($)	± 0.018	± 0.02	± 0.022	± 0.044

models used in this analysis supports this result. For geometry as simple as the cantilever spring, developing an FEA model to determine stiffness and fatigue properties is wasted modeling effort.

4 Addressing Model Prediction Uncertainty

In Section 3, utility theory was applied to the problem of model selection with good results. In this initial application of utility theory for model construction, model utilities were determined by measuring known differences in model prediction. This requires the construction of each model in the selection space prior to making the decision. In practice, the designer needs to select a best model prior to complete formulation and solution of the model. To allow selection before solution, estimation and uncertainty are introduced into the decision. Here, we extend the model selection procedure to explore the effects of uncertainty in model accuracy on selecting the best model.

When implementing the method presented here, the model's accuracy would be estimated and given some uncertainty bounds. Based on this estimation of model accuracy, the designer would follow the model selection procedure and select a best model. The estimation could be constructed as upper and lower bounds on an interval. For example, the model error (when compared to the best model or truth) is no less then 5% and no more then 15% of the predicted value. Another approach, which is considered here, is to represent model error through an estimated probability distribution. For example, the probability that the model is no worse than $\pm 5\%$ accurate is 33%. The probability that the model is no worse than $\pm 15\%$ is 67%. Also, because the error is coming from multiple independent sources, we assume that the probability distributions are normal.

With this in mind, we now have to consider how these uncertainties in model error affect the selection of the most useful model. Simply put, model prediction error uncertainties lead to attribute utility uncertainties. Combined with cost uncertainties, these lead to model utility uncertainties, and thus model selection uncertainties.

To determine our confidence in correctly selecting the best model, our approach is to propagate the uncertainties using the linearized uncertainty characterization relation [5]

$$\sigma^2(U)=\left(\frac{\partial U}{\partial x}\right)^2\sigma^2(x) \tag{4}$$

The approach explained here is presented for error propagation in design selection in [5]. It has been extended here for model uncertainty propagation.

The basic assumptions for Eq. (4) are that x is random and has a normal distribution. Equation (4) is obtained by making a first-order approximation while determining an induced distribution for U given the distribution for x [5,19]. The same equation is also obtained by deriving system variance using a Taylor's series expansion for system performance [20]. Third-order and higher terms are discarded in this method. The resultant uncertainty, assuming that the individual attributes are independent, can then be found by

$$\sigma^2(U_{Model})=\sum \sigma^2(U) \tag{5}$$

Continuing to treat our model error uncertainty as a normal probability distribution, we develop the relation between expected model utilities and model utility standard deviations

$$c=\frac{\varepsilon_1-\varepsilon_2}{\sigma_1-\sigma_2} \tag{6}$$

This number c is that required to stretch the two normal distributions until their areas of intersection are one standard deviation [5]. The expected utility of any model is represented by ε. Here σ_i is the resultant uncertainty of any model. The subscript 1 refers to the model having the best expected utility and the subscript 2 stands for any other model.

The numerical values for the cumulative distribution function $F(c)$ can then be found out for each model using the following relation:

$$F(c)=\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{c}e^{(c-\varepsilon)^2/2\sigma^2}dc \tag{7}$$

The probability that selecting the model with highest expected value over any other model is the correct decision is given by

$$Pr(decision)=2F(c)-1 \tag{8}$$

$Pr(decision)$ gives the value for the confidence in a selected model being better than the other model that is being considered.

Returning to the sway bar example, Table 4 shows model error estimates for each model abstraction. The error estimates are a difference in predicted performance from the best model. For example, in Table 4 the designer estimates that the stiffness predicted by Model 4 will be within 1.750 N/mm of the best model. These error estimates are treated as a 3σ upper and lower bound on model accuracy. In the examples worked in this paper, the model error distributions are assumed normal and centered at the expected values from Table 2. Treating the model error as a 3σ deviation translates as a 99.7% confidence that the model will provide a result within the range specified.

Using the method outlined above, these uncertainties are propagated to determine the uncertainties in the utilities. To simplify the example, linearized uncertainty characterization was assumed and linear curves were fitted to the individual utility functions in Figs. 12–15. The matrix of resulting utility uncertainties is shown in Table 5. The resultant uncertainties for the overall models can be found using Eq. (5). Then c, $F(c)$, and *Pr(decision)* are obtained as given in Eqs. (6)–(8). The values of σ, c, $F(c)$, and *Pr(decision)* are shown in Table 6.

Table 6 Resulting confidence measure calculations

	Model 1	Model 2	Model 3	FEA Model
Uncertainty in utility	± 0.537	± 0.508	± 0.145	± 0.044
C	0.4438	0.4537	n/a	5.51
$F(c)$	0.6715	0.675	n/a	1
Pr(decision)	0.343	0.35	n/a	1

Table 7 Data for confidence requirements of 25% and 50% certain

	Model 1		Model 2		Model 3		FEA Model	
	25%	50%	25%	50%	25%	50%	25%	50%
Uncertainty in utility	±0.919	±0.4115	±0.894	±0.4072	±0.032	±0.032	±0.044	±0.044
C	0.32	0.6825	0.32	0.6745	n/a	n/a	5.51	5.51
F(c)	0.625	0.7525	0.625	0.75	n/a	n/a	1	1
Pr(decision)	0.25	0.505	0.25	0.5	n/a	n/a	1	1

Table 8 Resulting sufficient model uncertainties for 25% and 50% confidence requirement

Uncertainty in *p* values	Model 1		Model 2		Model 3		FEA Model	
	25%	50%	25%	50%	25%	50%	25%	50%
Stiffness (N/mm)	±7.885	±3.155	±7.725	±3.075	±0.175	±0.175	±0	±0
Maximum stress (MPa)	±635	±270	±625	±270	±14	±14	±0	±0
Number of cycles	$\pm 30\times 10^3$	$\pm 15.4\times 10^3$	$\pm 28\times 10^3$	$\pm 15.3\times 10^3$	$1\pm\times 10^3$	$\pm 1\times 10^3$	±0	±0
Cost to model ($)	±4	±4	±4	±4	±7.5	±7.5	±9	±9

Table 6 shows the values for *Pr(decision)* (the probability of decision) that signifies the confidence measure for the certainty of the selection of Model 3 as the most useful model as compared to the other three models. That is, the probabilities of Model 3 being a better selection than Model 1, Model 2, and the FEA Model are 34.3, 35, and 100%, respectively.

5 Reverse Uncertainty Propagation

The model selection procedure and uncertainty estimation method detailed above provides a structured and rational framework for selecting a best model for engineering design and as such is a useful design aid. With the method developed, it is a simple matter to produce perhaps even a more powerful and useful design tool. Taking the above uncertainty estimation process and reversing it gives the designer a novel and powerful tool for model construction. By propagating acceptable decision uncertainties backward through the procedure, sufficiency requirements are generated on model cost and accuracy in performance prediction. In other words, the designer can specify an acceptable decision confidence level and, by performing a reverse uncertainty propagation, explore the required certainty of each of the models in the selection space.

For example, if the decision criteria mandate that confidence in the selection of the model with the best expected utility is better than the other models, it should at least be 25% for each model. This case is shown in Table 7. The resulting acceptable model errors are shown in Table 8.

If the designer wishes a selection confidence of approximately 50%, the model accuracy requirements increase. Shown in Table 7 are the data for a selection confidence of 50%. Shown in Table 8 are the resulting model sufficiency requirements. A review of Table 8 shows the increase in certainty requirements on performance prediction of the models needed to provide sufficient levels of model selection confidence.

The mapping from a particular confidence requirement to different model uncertainties is not one to one. Thus the designer can explore different reductions in uncertainty required to achieve the desired selection confidence. In the examples shown in Tables 7 and 8, the cost uncertainty has been left the same for each required confidence. The uncertainties in the FEA model were not adjusted as there is 100% confidence that Model 3 is the most useful model. The required uncertainty in estimating stiffness, maximum stress, and number of cycles is adjusted to yield the desired model selection confidence.

6 Conclusions

This paper formalizes the steps of model selection in engineering design through the use of utility theory and uncertainty propagation techniques. The methodology is illustrated through application to a simple design problem. We believe this work makes an important contribution to engineering design as it provides designers with the ability to select a best model. The importance of the model sufficiency estimation technique lies in the fact that it helps designers approach model selection as a rigorous mathematical procedure instead of relying on qualitative analysis based mostly on the designer's intuition. This work allows a designer to quantify the model selection process and thus makes a fundamental extension to model selection tools, such as that shown in Fig. 3. As a practical contribution, the method also provides the designer with important feedback on the confidence or certainty in having selected the best model.

In addition, in this paper a reverse uncertainty propagation method is presented that allows the designer to explore how truthful a model needs to be to allow for a desired confidence in selecting a best model. This important result allows a designer to quantify the situation in which, because of uncertainty in the validity of models, no model is clearly superior and engineering judgment (or a guess) is the best course of action. This situation parallels the middle box in Fig. 3 (the does-not-matter decision).

As a fundamental contribution, the work presented here moves us one step closer to being able to answer the question, how good does a model need to be? By exploring how uncertainties in the initial assessment of model accuracy and resolution affect the selection of the most useful model, this paper introduces an innovative approach for determining the accuracy requirement of a model. By using desired confidence measure specifications, the designer can determine the required predictive accuracy of a model. This method has wide applicability in engineering; it can help designers determine the required level predictive uncertainty of a model as well as other numerical design information, such as data samples and customer-need rankings.

The method includes the overall uncertainty or inability of a model to predict truth correctly and the cost of constructing that model. Not specifically addressed in the method are specific sources of model uncertainty, such as a model's sensitivity to parameter variation. By pinpointing different sources of uncertainty in a model, a designer would be better able to reason about which model is most useful or how to modify an inexpensive model to improve its truthfulness and make it the most useful model. Pinpointing specific sources of uncertainty in models is beyond the scope of this paper.

This work allows a designer to reason about model uncertainty. Nevertheless, remaining future work includes moving beyond simple reasoning about and dealing with model uncertainty to assessing model uncertainty. As it stands, the designer is still faced with estimating model truthfulness with little guidance. If the designer had available methods or knowledge to enable better estimations of model truth, then the method could be executed with high confidence measures. Developing a framework to enable the estimation of truthfulness of a general engineering design model remains future work.

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